**Part 1: Solve the following recurrences using substitution method.**

We can use the substitution method to establish either upper or lower bounds on a recurrence.

1. **T(n)= T(n-3) + 3 lg n.**

**Our guess: T(n)= O(n lg n)**

**Prove T(n) <= cn lg n for c > 0**

**For n=1**,

T(1) = T(1-3) + 3 lg 1

= -2 + 3(0)

= -2

**Inductive step:**

Upper Bound T(n) < =cn lg n for c>0

T(n)= T(n-3) + 3 lg n.

<= (cn lg n – 3) + 3 lg n

<= lg n ((cn – 3) +3))

<= cn lg n (for c >0)

**Therefore T(n) = O (n log n)**

1. **T(n)=4T(n/3) + n**

**Our guess: T(n)= O(n ^ lg3 4)**

**Prove T(n) <= cn^log3 4 for c > 0**

**For n=1,**

T(1)=> 4 ((n ^ log3 4)/3) + n

=> n(4/3 n^log3 4 +1)

This proves T(n) is not <= cn^log3 4

**Improved guess:**

Upper Bound T(n) <= cn^log3 4 - 3n c>0

**Inductive step:**

T(n)= 4T(n/3) + n

<= 4(c (n/3)^log3 4 – (3n/3)) +n

<= 4c (n/3)^log3 4 - 4n +n

<= 4c (n/3)^log3 4 - 3n

**Therefore T(n) = O (n ^ log3 4 - 3n)**

1. **T(n)=T(n/2) +T(n/4) + T(n/8) +n**

**Our guess: T(n)= O(n)**

**Prove T(n) <= cn for c > 0**

**For n=1,**

T(1) =T(n/2) +T(n/4) + T(n/8) +n

= 1/2 +1/4 +1/8 +1

= 23/8

**Inductive step:**

Upper Bound T(n) <= dn

T(n)= T(n/2) +T(n/4) + T(n/8) +cn

<= dn/2 +dn/4 +dn/8 +cn

<= dn(7/8) +cn

<= n (7d/8 + c)

d(7/8) <= 0 which is therefore c >= +d ⅞

T(n)=O(n)

Lower Bound T(n) >= dn

T(n)= T(n/2) +T(n/4) + T(n/8) +cn

>= dn/2 +dn/4 +dn/8 +cn

>= dn(7/8) +cn

>= n (7d/8 +c)

d(7/8) +c >= 0

Therefore T(n)=Ω(n)

1. **T(n)=4T(n/2)+n^2**

**Our guess: T(n)= O(n^2)**

**Prove T(n)<=cn^2 for c > 0**

**For n=1,**

T(1) = 4T(n/2) + n^2

= 4(1/2) + 1^2

= 3

**Inductive step:**

T(n)= 4T(n/2)+n^2

<= 4 c (n/2)^2 + n^2

<= c n^2 + n^2

<= n^2 (c+1)

This proves T(n) is not <= cn^2

**Improved guess**

T(n) < =cn^2 - n , c>0

T(n) = 4T(n/2) + n

<= 4(c(n/2)^2 – (n/2)) + n

<= cn^2 – 2n + n

<= cn^2 – n

**Therefore T(n) = O(n^2)**